

This Month's Problem...

Here's a problem that we have been thinking about on the CLIME bulletin board. It's called the Zero-Sum problem. It appeared in the *Computing Teacher* (2/1990) along with a BASIC solution. I would like to see one in Logo. Here's the problem:

Consider the sequence of digits for 1 through N in increasing order (1, 2, 3, 4, 5, ...N). Insert either a (+) for addition or a (-) minus for subtraction in the space between each pair of digits. Now sum the result and see if you get a zero. Write a program that will find all sequences of length N that produces a zero sum.

We look forward to your solutions. If you have access to a modem enter your solution on the CLIME bulletin board. Contact Sandy Dawson c/o Simon Fraser University, Dept. of Mathematics Education, Vancouver, BC for an account. He can tell you how you can access the bulletin board for only the cost of a local call.

Inspi...continued from page 11

```
TO TOTALTURN :ANGLE :INC :TT
  INSERT (WORD :ANGLE " ; :TT " ; CHAR 32)
  MAKE "ANGLE REMAINDER :ANGLE + :INC 360
  MAKE "TT REMAINDER :ANGLE + :TT 360
  TOTALTURN :ANGLE :INC :TT
END
```

It was hard getting the first line (the insert line) to do what I wanted. On the other hand, when I started to concoct the procedure, I wasn't sure I could do it at all with LogoWriter commands. Here is the printout for TOTALTURN 0 30 0 to accompany Figure 1 (where the turtle starts off with angle = 0):

TOTALTURN 0 30 0

0,0; 30,30; 60,90; 90,180; 120,300; 150,90; 180,270; 210,120; 240,0; 270,270; 300,210; 330,180; 0,180; 30,210; 60,270; 90,0; 120,120; 150,270; 180,90; 210,300; 240,180; 270,90; etc...

We can read from the printout the sequence of totalturns for the successive steps of the turtle:

0, 30, 90, 180, 300, 90,

I have placed these values in Figure 1 next to the corresponding turtle steps on the way out to point P. Continuing, the totalturns are:

270, 120, 0, 270, 210, 180

which brings the turtle back to point 0. These return values are UNDERLINED in figure 1. Since the turtle retraces its path exactly, the two TT values associated with a turtle step should differ by 180. Inspection of these values in figure 1 shows this to be true.

But is it just an accident in this particular example that at every turtle step, the associated values differ by 180? We shall see. Let

θ be the initial angle, so that we are running INSPI :SIDE : θ : θ . Then the TOTALTURN procedure would produce the following:

0,0; θ , θ ; 2 θ ,3 θ ; 3 θ ,6 θ ; 4 θ ,10 θ ; 5 θ ,15 θ ; ...

Suppose a multiple of θ is reached which equals 180 or an odd multiple of 180 (at which point the turtle reverses its direction.) Let α be a multiple:

$\alpha = n\theta = \text{odd multiple of } 180$

Let β be the totalturn corresponding to α , so that a part of the total turn procedure printout would be:

...; $\alpha - 3\theta$, $\beta - 3\alpha + 3\theta$; $\alpha - 2\theta$, $\beta - 2\alpha + \theta$; $\alpha - \theta$, $\beta - \alpha$; α , β ; $\alpha + \theta$, $\alpha + \beta + \theta$; $\alpha + 2\theta$, $2\alpha + \beta + 3\theta$;

obtained by working right and left from " α , β ".

Note that $\beta - \alpha$ and β are TT values associated with the step that ends at P. Now we can verify that two TT values associated with a turtle step differ by an odd multiple of 180:

$\beta - (\beta - \alpha) = \alpha = \text{odd multiple of } 180$

$(\alpha + \beta + \theta) - (\beta - 2\alpha + \theta) = 3\alpha = \text{odd multiple of } 180$

$(2\alpha + \beta + 3\theta) - (\beta - 3\alpha + 3\theta) = 5\alpha = \text{odd multiple of } 180.$

etc...

Thus if the turtle ever reverses its direction (given that ANGLE = INC initially), it will retrace its path exactly. This disposes of the type of pattern in figure 1 where the multiples of θ contain an odd multiple of 180. But there are values of θ whose multiples do not include an odd multiple of 180. This can happen if these multiples include 360 but not 180, as for $\theta = 8, 120$, and other angles. Such values of θ give the type of pattern in figure 2. Is there possibly a third type? I don't think so, but I don't know.



Conjecture: Given that ANGLE = INC in INSPI :SIDE :ANGLE :INC, either the turtle does an about face somewhere and retraces its path, or it trails off to infinity winding around a straight line.