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produce non-simple ones.

About this time some groups started to ponder the idea of inputting fractional or even irrational angles in POLY.EXPERIMENT. However, other groups were very hesitant; it seemed foreign to them to let the computer experiment with non-integer inputs. This was surprising because whenever we were doing mathematics without the computer the question of rational inputs was never questioned, the mathematics demanded them.

Reflecting back on this experience, we found it hard to resist the temptation of lecturing on the appropriate mathematics that would make many of the independent investigations no more than immediate corollaries of some well know theorems. Instead we forged ahead, answering only specific questions the class asked. The only mathematics we lectured on at this time were some simple ideas in number theory and least common divisors. We did spend time however on learning how to be critical observers and be good data recorders.

The mathematics that now took place was too good to be true. We had each group write up all their conjectures about POLY.EXPERIMENT. The first submission of conjectures by our class was a shock to us until we realized that this was the first time that our mathematics students were ever asked to create mathematics. Their conjectures for the most part did not make sense, or if they were written so they were understandable, they were not decidable statements like "Inputs that are multiples of 48 create pretty pictures".

After we asked our class to resubmit their conjectures we took a subset of 25 of these and wrote them on dittos for everyone to have. Some of these conjectures are in figure 2.

Conjecture 1: A regular simple n-gon is formed when the input angle is 360/n or 360 - (360/n).

Conjecture 2: If 360 / input angle is rational and not an integer, then a non-simple star polygon is formed.

Conjecture 3: If the input angle is of the form (n-2)*180 / n where n >= 5 and odd, then a 2n-star polygon is formed.

Conjecture 4: If n ε Z then a regular simple n-gon is created by POLY.EXPERIMENT with an angle input of 360 / n.

Conjecture 5: If an angle input of A produces an n-gon, then an input angle of 180 - A, will produce either an n-gon, a 2n-gon, or n/2 -gon.

Figure 2

An entire class period (two hours) was spent interpreting these conjectures. The creators had to answer any questions other classmates may have had about the interpretation of their statements. No one was asked to discuss the validity of any of the conjectures, just their meanings (for instance, Conjecture 4 is obviously false if n = 0.) The class was next assigned the task of deciding which conjectures were false and if a simple correction was possible (like conjecture 4), correct it. For the conjectures that they believed to be true, they were asked to make an implication tree which would establish the logical connections between the conjectures. After compiling their results, we observed that collectively, they had any conjecture implying any other conjecture, including all the false ones!

After our initial frustration with the above absurdity, what transpired next was one of the most fruitful parts of the class. With students leading the discussion we got concrete examples of converses, contrapositives, etc.; our students were certainly exposed to these concepts before, but clearly had never internalized them. Students would come to class with "what if" on their minds. Discussion like the following abounded in our class: "What if Conjecture 8 was true... One group would respond with "that would imply Conjecture 11 is also true because it s a special case of 8". We let the class discuss (and argue) over these implications for several days before we interjected any definitive ideas about the topic.

The ideas discovered by our class were not "new" results. Actually most of them followed from some well known results in "Turtle Geometry" (DiSessa, 1980): The Total Trip Theorem which states that if the turtle is ever going to return to its original position with the original heading then its total turning must be an integer multiple of 360. And the Poly Closing Theorem which states that the converse of the Total Trip Theorem is true for the procedure POLY.EXPERIMENT.

We presented these theorems to allow them to prove the conjectures that they had created. (The implication tree that they finally agreed upon showed that there were only 4 conjectures out of the original set that actually needed verification.)

Reference

Abelson, H. and DiSessa, A. *Turtle Geometry,* Cambridge, MA: MIT Press (1980).

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Figure 2

figure so that the figure, as a whole, is symmetric about the center. I was particularly interested in how it could be that the turtle would retrace its path exactly. In the other pattern, illustrated in Figure 2, the turtle trails off to infinity winding around a straight line.

To help me in analyzing these patterns, I wanted some numerical evidence of what was going on. I wrote a procedure to print out the values of angle and TOTALTURN at each step. I learned about TOTALTURN on pages 24 to 39 in Abelson and diSessa's wonderful book, Turtle Geometry (MIT Press, 1980). Here is my procedure:

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