The Random Turtle: A Logo Simulation of Two Classic Problems

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A powerful way for students to develop a sense for the probability of a particular outcome of an experiment is for students to actually perform the experiment, or to simulate it if it is tedious or difficult to perform. The Logo turtle has at her disposal a random number generator that allows her to simulate probabilistic events. We will look at two simulations.

The Birthday Problem

What is the probability that, in a room containing 30 people, there are two or more people with the same birthday (month and day)? This situation would be difficult to test experimentally; we can, however, simulate it using Logo. The command **RANDOM *number*** returns a random integer between 0 and (number -1), so we can use the expression **1 + RANDOM 366** to get a random number between 1 and 366. (The assumption is that all 366 days are equally likely to occur; that is almost correct.) Using a Logo procedure called **BIRTHDAY**, a room filled with 30 people can be simulated.

```
BIRTHDAY     295 124
265 301      52 228
295 74       327 267
191 162      150 316
34 72        129 227
127 23       131 134
6 269        263 66
198 136
362 38
```

In this simulated room, birthday number 295 is repeated. Twenty runs of this procedure produce *fourteen* rooms with repeated birthdays, a result that flies in the face of intuition. Theoretically, the odds are quite good that there will be repeats. To see this, find the probability that there will be *no* repeated birthdays.

$$P(\text{No Repeats}) = \frac{366}{366}\frac{365}{366}\frac{364}{366}\cdots\frac{337}{366} \approx .295$$

Repeated Birthdays) = 1 - P (No Repeats), so
P(Repeated Birthdays) = 1 - .295 = .705

This article was presented as a session at the NCTM Annual Meeting in Orlando ,Florida last April 14, 1989. The procedures are written in Terrapin Mac Logo and are enclosed separately.

The Logo procedure BIRTHDAY is, therefore, a simple way to challenge the intuition of younger students and set up the need for this theoretical explanation.

```
TO BIRTHDAY
REPEAT 15 [PRINT (LIST 1 + RANDOM 366 1 +
RANDOM 366 ) ]
END
```

Buffon's Needle Problem

The historical background and theoretical solution of this problem were taken from A History of π, by Petr Beckmann (Golem Press, 1971).

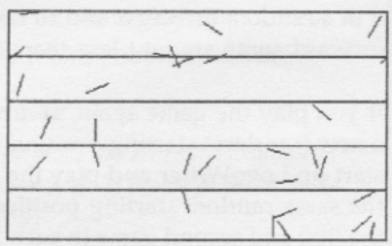
In 1777, George Louis Leclerc, Comte de Buffon, proposed the following problem. Suppose that a needle of length L is thrown onto a horizontal plane surface ruled with parallel lines spaced apart by a distance d (d>L). What is the probability that the needle will intersect one of the parallel lines?

This problem can be simulated using Logo. The turtle can draw the grid of parallel lines and randomly place needles on the grid. She can also count the number of needles that hit the grid lines. Three different approaches will be taken to this simulation, each appropriate to students at a different level.

Simulation 1: The Logo procedure BUFFON first draws a series of horizontal grid lines. The needles are then placed, a random location for the center of the needle is determined, the turtle is placed there, and a random direction is chosen for the turtle's heading. The turtle then walks forward and back to draw the needle.

```
TO BUFFON :NUM
DRAW.GRID
DROP.NEEDLES :NUM
HT
END

BUFFON 20
```



This simulation will allow students to throw needles at grids and count the number that hit lines. The probability is then just the ratio of "hits" to the number of needles thrown. Of course, students can actually do this with toothpicks and grid paper or with chopsticks on a tiled floor. The advantage is that a number of trials can be made by each student (and you do not have to clean up the classroom afterwards.)

Continued next page