King Arthur Discovers Logarithms

Finding a more "definitive" rule for the knights problem

The rule that solves the knights problem is a bit cumbersome at best. It would be nice to be able to write this rule and get the results in a more direct way. As it turns out, this is possible, but we have to use a bit more mathematics.

Lets look at the pattern again in a slightly different way.

For 1 knight the winning seat is $2(K-2^0)+1 = 1$
For 2 knights the winning seat is $2(K-2^1)+1 = 1$
For 3 knights the winning seat is $2(K-2^1)+1 = 3$
For 4 knights the winning seat is $2(K-2^2)+1 = 1$

and so on. We can generalize and say the following.

If there are K knights at the round table, the winning seat will be $2(K - 2^P) + 1$ where P is the largest power of 2 where 2^P is the largest value that does not exceed K. So now the problem becomes finding P. To solve this we are going to take advantage of some things we know about logarithms.

Some Background on Logarithms

A logarithm is just another name for an exponent. For example, the logarithm of 2^P is P to the base 2. Below is a table of binary numbers and their logarithms for base 2. For example, the logarithm of 128 (base 2) is 7, since $128 = 2^7$.

No.	Log	No.	Log
1	0	64	6
2	1	128	7
4	2	256	8
8	3	512	9
16	4	1024	10
32	5	2048	11

Now let's say we wanted to multiply 32 x 64 using this chart. We can write these numbers in exponential form and multiply them as follows.

$$2^5 \times 2^6 = 2^{5+6} = 2^{11}$$

Note that the product of these numbers is 2 to the 11th power which is the *sum* of the exponents. Now looking at the chart above 2048 is listed as the number that has 11 as an exponent. This is also the product of 64 and 32.

What made logarithms useful is that they can turn multiplication problems into addition problems. This was such a useful idea that John Napier developed a table of logarithms that was used to ease the drudgery of multiplication and division. Napier's table used 10 as a base rather than 2. So, in base 10, the logarithm of 1 is 0 ($10^0=1$) and the log of 10 is 1 ($10^1=10$) and the log of 100 is 2. Below is a list of logarithms for numbers between 1 and 10. Notice that $2 \times 3 = 10^{.301} \times 10^{.477} = 10^{.778} = 6$. It turns out that any multiplication or division problem involving whole numbers could be handled in a similar way. In fact, you can experiment with this idea using a calculator that has a logarithm key. Understanding how logarthms works also explains how a slide rule functions. (It is an interesting and useful activity to actually make a slide rule using pieces of cardboard. See Jacobs.)

No.	Log
1	.000
2	.301
3	.477
4	.602
5	.699
6	.778
7	.845
8	.903
9	.954
10	1.000

Back to our problem of finding a better rule for the King Arthur's Dilemma problem. We want to find a P value for any value of K (# of knights) that will deliver W (the winning seat). For example, if K = 28, then the formula for the winning seat becomes $2(28 - 2^4) + 1$. But how can we find the number 4 from 28? It turns out that 4 is the logarithm in base 2 of the number we want to subtract. But how can we find, in general, logs of a base 2 number? The log tables and calculators give base 10 logs. There is a way. Let's

Continued on next page