

look at the problem again. We want to find X when $2^x = 28$.

We know the solution lies between 4 and 5. If $2^x=28$, then $\log_{10} 28$ must equal the $\log_{10} 2^x$. (Note: $\log_{10} X$ is assumed when $\log X$ is written. These are called common logarithms.) We can find the log of 2^x by noticing that finding this logarithm is the same as finding the log of each 2 in the product and adding it X times (assuming, of course, that X is an integer. In other words,

$$\begin{aligned}\log 2 \times 2 \times 2 \dots \times 2 \text{ (X times)} \\ &= \log 2 + \log 2 + \dots + \log 2 \\ &= X \log 2.\end{aligned}$$

(If the previous line isn't clear, review how logs were used in the binary examples above in turning multiplication problems into addition problems.) This means that we can rewrite $\log 2^x = \log 28$ as $X \log 2 = \log 28$. Solving for X, we have $X = \log 28 / \log 2$ which can be determined from a table or a calculator.

$$\begin{aligned}\log_2 28 &= \log 28 / \log 2 \\ \log 28 &= 1.447 \\ \log 2 &= .301 \\ \log_2 28 &= \log 28 / \log 2 = 4.807\end{aligned}$$

Back to our original problem. For 28 knights the largest binary number less than 28 is 16 which has a logarithm of 4 (base 2). We now have a method for finding the logarithm of 28 (base 2). Now that we know how to find the logarithm of K (base 2), can we determine the appropriate binary number to subtract? In this case, the logarithm of 28 (base 2) is 4.807, and the binary number we're looking for is 16 (2^4) with a logarithm of 4. Notice that if we round down the logarithm of K to the nearest whole number, we have the desired value for P. So what does this look like in formula form? A bit complicated.

$$2^{(K-2^{\lfloor \log_2 K \rfloor})} + 1$$

where K is the number of knights. ($\lfloor k \rfloor$ means to round down the value of k to the nearest whole number.)

Here is the program that will output the winning seat for a given number of knights written in Logo.

```
TO TABLE :K
MAKE "P INT (LOG :K) / (LOG 2)
MAKE "W 2* (:K - POWER 2 :P) + 1
OUTPUT :W
END
```

SHOW TABLE 53

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NOTE: If the Logo you are using does not have a common logarithm primitive, the problem of having the computer determine the winning seat is a bit more challenging.

References

Jacobs, Harold. *Mathematics: A Human Endeavor* (W.H. Freeman & Co., New York)

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much brighter by providing us with "state of the art" projection system which he borrowed from his school. Jim didn't want us to forget that you can get a lot of mathematical exploration mileage out of Logo without a lot of programming - which is an issue with many teachers. He shared examples of simple procedures that produced star patterns. In his classes he asks his students to see what kinds of conjectures can be made from this. (Editor's note: Similar examples can be found in our Microworld collection. See pages 6 & 16. Titles include: Spiros, Billiards, Exploring It, and Geoshapes. Also see Rectangle Lab on page 4.)

Henri Picciotto (Urban School in San Francisco) shared a personal odyssey of his involvement with using tools in teaching mathematics. In his formative teaching years with Logo he helped students create Logo tools that they used to solve problems. He found this very effective, but was disappointed that his students did not incorporate his procedures into

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