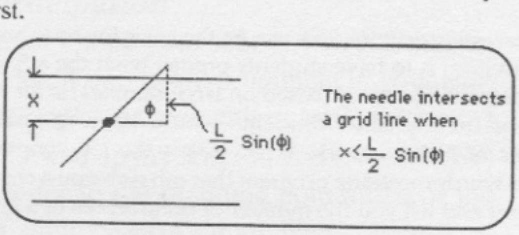
Random Turtle...continued from previous page

Another advantage of having students write, or at least understand, this simulation is that the method of placing the needles is the key to the solution of the problem.

In order to see the relationship between the length of the needles and the number of the hits, **NEWBUFFON** allows the student to choose the length as well as the number of needles thrown.

Simulation 2: To get the turtle to count the hits, some trigonometry is needed. The turtle must calculate the vertical distances from the center of the needle to the nearest grid line, and from the center of the needle to the end of the needle. A hit occurs if the second distance is greater than, or equal to, the first.



The procedure **BUFFON2** counts the number of hits by checking for a hit each time it places a needle on the grid. It then prints the number of hits and probability of getting a hit, where

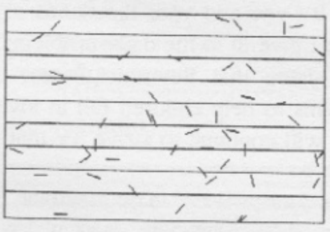
$$P(\text{Hit}) = \frac{\text{Number of Hits}}{\text{Number of Needles}}$$

Running **BUFFON2 100** twenty times produced the results summarized in the stem-and-leaf plots and illustrated below:

```
2 |          where 3|4 means 34 hits
• | 6 7 8 8 9 9
3 | 0 1 1 1 2 3 3 3 4 4
• | 5 8 8
4 | 2
• |          Median = 31.5
```

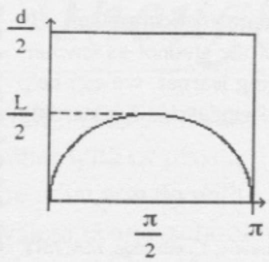
```
BUFFON2 100

32 HITS
PROBABILITY IS
ABOUT .32
```



Simulation 3: Leclerc solved this problem theoretically using calculus. He noted, as described above, that a needle intersects the line when the distance from the center of the needle to the nearest grid line is less than the vertical distance from the center of the needle to its point.

Since the position of the needle depends on two variables, the location of the center and the direction of the needle, he represented all possible positions of the needle as the points contained in a rectangle with dimensions π and d/2. (The center of the needle can be at most d/2 units from the nearest grid line, and the angle of the needle could vary between 0 and π. (See below.) Then a hit will occur if the needle's position point lies under the curved graph of the equation $y=(L/2)\sin(\phi)$. The probability of a hit is the ratio of these areas.



$$P(\text{Hit}) = \frac{\int_0^{\pi} \frac{L}{2}\,\text{Sin}(\phi)\,d\phi}{\frac{d}{2}\,\pi}$$

$$P(\text{Hit}) = \frac{2L}{\pi d}$$

LaPlace extended this result in 1812 to state that repeated trials of this event should therefore give an approximate value of π. (He was the first mathematician to use this "Monte Carlo" technique.) From the equation above,

$$\pi = \frac{2L}{d\,P(\text{Hit})}$$

and in the special case where the length of the needle, L, is half the distance, d, between grid lines (as in our simulation.)

$$\pi = \frac{1}{P(\text{Hit})} = \frac{\text{No. of needles}}{\text{No. of hits}}$$

The procedure **BUFFON3** will print this approximation of π. For example, when **BUFFON3 100** was run 20 times, the turtle produced the following results:

```
2 |                   where 3|2 means π is approx. 3.2
• | 5 6 9 9 9 9
3 | 0 0 0 0 1 1 2 3 3
• | 6 7 7 8           Median value of π is 3.077
4 |
• | 8
```

The 4,000 trials summarized in the stem-and-leaf plots above can be combined to give a total of 1275 hits and an approximate value of 3.137.

Discussion

This problem was investigated at three levels: first, simulations of the original problem that could be written and used by middle-schoolers; second, an improvement that used trigonometry; finally, a full development using calculus. Younger students gain experience with a situation in which a given number of needles will not always produce the same number of hits. However, the number of hits will cluster

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