***Quarts, Pail, or Pitcher**...continued from page 2*

Next, the promised unwinding of Euclid:

```
TO FIRST.COEFF :B :A
IF (MOD :B :A) = 0 [OP 0]
OP SECOND.COEFF:A (MOD :B :A)
END

TO SECOND.COEFF :B :A
IF (MOD :B :A) = 0 [OP 1]
OP (FIRST .COEFF:A (MOD :B :A)) - (SECOND.COEFF
:A (MOD :B :A)) * (QUOT :B :A)
END
```

Here, MOD is REMAINDER (fixed so it acts right on negative inputs), and QUOT (X,Y) outputs the whole number quotient from X/Y. In other words, if Y > 0, X = QUOT(X,Y)*Y + MOD(X,Y), and 0 <= MOD(X,Y) < Y. You can check that GCD(930,42) = FIRST.COEF(30,42)*42. It will be clear why these functions work if you do a few (dozen) hand calculations. A crude solution to your problem (with no error checking is SOLVE:

```
TO SOLVE :A :B :T
OP (LIST (QUOT :T :A) LIST (MOD :T :A) / (GCD :B
:A) * FIRST.COEFF :A :B (MOD :T :A) / (GCD :B :A)
* SECOND.COEFF :A :B)
END
```

For example, SHOW SOLVE 30 42 48 will produce [1 [9-6]]. This means that to get 48, fill the small pail 9 times, empty the big pail 6 times, and "spice" once with the small pail.

Clearly, this is only a germ of a solution. I bet that finding the minimum number of transfers amounts to rewriting FIRST.COEFF and SECOND.COEFF so that they aren't so stupid. This has been a nice project for my students!

Regards,
Al Cuoco
Woburn HS
Woburn, MA 01801

On the Saxon controversy

Dear Editor,
Saxon speaks eloquently for the student who finds mathematics difficult. He proudly points to his own personal history of failure in math, and blames the "mathematics establishment" for it. He proposes three solutions: (1) A spiral curriculum, with much review of concepts and techniques at all times, and preview of difficult topics as early as possible. (2) Minimize obscure notation and terminology. (3) Reliance on rote memorization of dozens of algorithms, with no attempt at understanding; avoidance of non-routine problems. His first two proposals are excellent. The approach embodied in the traditional textbooks is to start with difficult jargon and notation, deal with a topic until the end of the chapter, and then move on to something else. The student who never mastered it breathes a sigh of relief; the student who did prepares to forget; and the teacher hopes to do a better job next year. This is no way to get difficult ideas across. No matter what textbook you use: preview and review must be woven into your course; notation and terminology should help, not obstruct understanding; and every test should be a final. This approach makes it possible to demand mastery from all or almost all of your students.

Saxon's third and crucial proposal, on the other hand, is dead wrong. It is what Ihor was reacting to, I think, and I agree with his indignation. Let's face it, when faced with a student who has trouble learning a concept, it is easier to give up and tell them to memorize some microskill which will get them through the day than to get to the bottom of the problem and help them really learn it. But the only way to gain mathematical power is through creating mathematics, not parroting ill-understood algorithms. The beauty and power of mathematics, and the ability to solve non-routine problems, are accessible to all students if they are given opportunities to think, talk, and experience mathematics.

Sincerely,
Henry Picciotto

See Profiles (page 4) for Henry's address. Δ

***Innumeracy**...continued from page 5*

For example, many people cancelled flights to Europe because they feared acts of terroism even though the probability of such an event is miniscule. Also, people continue to smoke cigarettes not realizing how they increase their chances of living a shorter life. "If the chances of a plane crash were the same as a person's chances of dying from smoking then we would have 3 jumbo jets crashing every day." Paulos says. This is just one example from a book filled with many examples of how having some intutive number sense would help people to be better informed and thus lead more productive lives.

The book presents its case for the need for doing something about innumeracy. Paulos shares many reasons for why innumeracy develops, and offers a few suggestions about what can be done about this problem. Some suggested solutions are: (1) The media should be used to help dispel misconceptions about number sense. (2) Elementary

*See **Innumeracy**...page 22*