The Logo Corner - Turtle Algebra

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The concept of function is a crucial idea — some argue that it is the most important idea — underlying school algebra. Logo procedures can be thought of as functions. For example, the procedure

```
TO SQUARE :L
REPEAT 4 [ FD :L RT 90 ]
END
```

takes a number as input and returns a square with sides equal in length to that number. It is a function mapping the set of real numbers onto the set of squares. Here are two Logo-based algebra activities that I have used with students.

Function Machines

The following Logo procedures represent the functions f(x) = x + 5 and g(x) = 3x.

```
TO F :X
OUTPUT :X + 5
END

TO G :X
OUTPUT 3 * :X
END
```

I have used "function machines" like these to play a "Guess My Rule" game with students in both general math and algebra classes. With a single computer and projection device at the front of the room, and with a bunch of these machines already defined, I ask students to gather data in order to guess the rules. They can only make two kinds of statements: they can either ask me to input a number, or they can state a conjecture about what would result if I input a particular number. When they are able to predict, they are to write their rules both in words and using symbols. The projected image resembles that shown below.

```
? F 5
RESULT: 10
? F 8
RESULT: 13
```
(Data from Machine F.)

Several important mathematical ideas come out of classroom interactions as this game is used periodically throughout the school year. For example, students learn to summarize rules like "take the input number and add five" (for the machine F) using symbols like Out = In+5, and, eventually, y=x+5.

Students also have opportunities to compare equivalent symbolic summaries. For instance, after gathering data like that shown below, my students offered two rules for the machine named H: "Multiply the input by two and add one," and "Add the input to the input plus one." This led to a discussion of the equivalence of the expressions 2x + 1 and x + (x + 1).

```
? H 3
RESULT: 7

? H 5
RESULT: 11

? H 10
RESULT: 21
```
(Data from Machine H.)

When I type the string of characters F G 5, very few of my students predict the result of 20, which is the composition f(g(5)). Many expect a result of 150, which they get by multiplying f(5) and g(5). However, after about ten minutes of data gathering and heated discussion (little of which includes me!), most students are able to predict, for example, that the result of the command F G F H 2 will be 35.

The Box that Holds the Most

If you cut squares from the four corners of an 8.5-inch by 11-inch sheet of paper, and then fold up the resulting tabs, you get an open- top box. How large should the cutout squares be in order to create the box that holds the most? Calculus students, all of whom get to do a problem like this, never get to examine the boxes themselves to see how they

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