Logo Microworlds: Investigating Right Triangles and the Pythagorean Theorem

by John Olive

In this microworld students start with two procedures RT.TRI and LT.TRI which create right and left oriented 90 degree triangles. Using these procedures, students can investigate relationships among right triangles, rectangles, squares, rhombi and general parallelograms. Using the inputs to figure the length and breadth of various shapes can lead to an understanding of a model for the measurement of area.

After investigating the area of various rectangles using the right triangle procedures, students can be introduced to the next challenge which uses a new procedure, SQ.HYP. SQ.HYP uses the same inputs as the right triangle procedures to create a figure used by Bhaskara, a Hindu mathematician of the 12th century to prove the Pythagorean Theorem (see Figure 1).

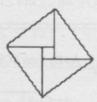


Figure 1.

The figure consists of 4 congruent right triangles arranged so that their hypotenuses form the sides of the square containing the four triangles. By realizing that the inputs to SQ.HYP are the lengths of the legs of each triangle in the figure, students can calculate the total area of the figure. Trying this activity with a variety of inputs and comparing the figures produced can lead to the students' own discovery of the Pythagorean Theorem. A look at the text of the triangle procedures and the subprocedure HYPOTENUSE would illustrate a powerful application of the Pythagorean theorem! My students were also intrigued by the ARCTAN function (used in the triangle procedures) which led to an informal (but informative) discussion of the trigonometric ratios in right triangles.

INSTRUCTIONS:

To create right angle triangles use the procedures RT.TRI and LT.TRI. Both procedures use the same inputs.

Find out what parts of the triangles use each of the inputs.

Experiment with RT.TRI and LT.TRI. Can you make a rectangle wth them? What about a square rectangle?

Next, find out what the procedure SQ.HYP does. It uses the same inputs as the triangle procedures.

What relationship can you discover about the area of the whole figure and each of its parts?

PROCEDURES:

TO RT.TRI:B:L
FD:BRT90FD:L
LT ARCTAN(:B/:L)
BK HYP:B:L
LT90 - ARCTAN(:B/:L)
END

TO LT.TRI:B:L
FD:BLT90FD:L
RT ARCTAN(:B/:L)
BK HYP:B:L
RT90 - ARCTAN(:B/:L)
END

TO HYPOTENUSE :BASE :LEG
OP SQRT (:BASE * :BASE + :LEG * :LEG)
END

TO HYP :BASE :LEG OP HYPOTENUSE :BASE :LEG END

TO SQ.HYP:B:L
SETBG 6 HT
REPEAT 4 [LT.TRI:B:L FD:B RT 90 BK:L]
END

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If you try to do these procedures with LogoWriter 1.1, you will discover that there is no arctangent primitive. However, you can write a procedure ARCTAN that will work reasonably well. See page 10.