## The Handshake Problem, Part 2

by Alan Ozer

Another way of looking at the Handshake problem is geometrically. We can let points on the circumference of a circle represent "handshakers" and lines connecting the points "handshakings".

Points	Lines	0	
3	2	0	1
4		^	43
5		(	40
etc.		0	

Connecting the points creates attractive designs especially if colored pencils are used. You can also do curve stitching with nails, wood, and thread.

Let's see if we can modify the handshake procedures from the previous article to draw geometric patterns on the computer. Instead of entering names of people, we will enter the number of points. Then the points will be arranged symmetrically on a circle, and each point will be connected to every other point. Also, we will enter the length of the distance between the adjacent points. Here is a general plan for the program:

CONNECT connects lines to all the points.

CON picks out the first point on the list.

CON1 draws and connects the first point to all the others.

POLYGON draws a polygon and stores every point on

that polygon in a list.

SAVEPT saves the location of each point in the list POINTS.

CONNECT prompts you for a number of points and length of each side.

TO CONNECT
MAKE "POINTS [ ]
PR [TYPE IN HOW MANY POINTS]
MAKE "NUMOFPTS FIRST REQUEST
PR [TYPE IN THE LENGTH OF EACH SIDE]
MAKE "LENGTH FIRST REQUEST
MAKE "TOTAL 0
POLYGON :NUMOFPTS :LENGTH
MAKE "C COUNT :POINTS
CON :POINTS 2
END

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## Another way of looking at the Handshake problem

People or points	Handshakes or lines	Sums
2	1	1
3	3	1+2
4	6	1+2+3
21	210	1+2+3++20

Notice that you can figure out the number of shakes by summing up the consecutive integers from 1 to the number of people minus 1. Karl Gauss, a German mathematician, is credited with discovering a shortcut for adding consecutive integers. Let's say we want to add the first 20 counting numbers. Start by matching the first number (1) with the last counting number in the list (20) and add them together (21). Next, match the second number (2) with the next to last number (19) and add them up (21). Notice the answer is the same. In fact, if you continue the process, you will have 10 pairs of numbers, each pair adding up to 21. So the total sum is 10 times 21 or 210.

We can generalize this result: If N is the largest counting number, then the sum of all the counting numbers up to N is N/2 (N + 1). Modifying for handshakes, the formula becomes H = P (P-1)/2 where H = handshakes, P = Number of People. This can be put into a Logo procedure.

TO SHAKES :PEOPLE
OP :PEOPLE \* (:PEOPLE-1) / 2
END

PR SHAKES 21 210