In the Spirit of Eratosthenes

Measuring the Circumference of the Earth

By Ihor Charischak

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Back in 1972, I came across an issue of *The Mathematics Teacher* with an article that described one teacher's effort to collaborate with another school in an attempt to re-create and confirm the results that an ancient librarian had once come up with. The librarian's name was Eratosthenes, and he quite accurately measured the circumference of the earth from Alexandria, Egypt, in approximately 200 B.C.

I was inspired to try the activity with a second-year algebra class that I was then teaching. I attempted to involve two schools—one in Michigan and the other in Florida—but nothing materialized.

Fast forward to 1995. While creeping along the Internet (I had not yet discovered surfing), I read that a high school mathematics teacher in Illinois was hosting something that she called the Noon Observation Project. It turned out to be a worldwide collaboration among schools who sought to re-create what Eratosthenes had done so long before. Because the experiment requires participants to measure shadows at about the same time (when the sun is at its highest point in the sky), "real-time" communication is extremely important. E-mail turned out to be a great way to do this.

I found out later that the project dated back to at least 1988 when Jim Levin at the University of Illinois, with the help of Al Rogers and his ubiquitous FrEdmail network, made such an experiment practical. Because I was working mostly with elementary and middle school teachers in workshops that I lead at the center for Improving Engineering and Science Education, I was tempted to get involved but held off, thinking that the use of trigonometry would be a barrier for many of my teachers. So I put this project on the back burner.

A year later, while visiting my favorite "library," Barnes and Noble, I came across a children's book with an intriguing title, *The Librarian Who Measured the Earth* by Kathryn Lasky, and read it while standing in the aisle. (One of the delights of children's books is their brevity.) The book was wonderful, but I did have one problem with it: All of the mathematics were on one page and probably beyond the understanding of the intended audience. This sparked an idea and a challenge. I wanted to explain the mathematics behind this story in a way that children could understand. Little did I know what I was getting myself into.

The process of making the mathematics comprehensible took me on a fascinating learning journey. One thing I learned is how important good physical models are in representing real-world phenomena. Concepts such as how the earth revolves around the sun seem simple at first glance, but they can be quite hard to conceptualize, unless you have something to look at and turn in your hand. Language is another hurdle. I've read many descriptions of the phenomena that I describe here, and most are intended for an older audience. My challenge was to explain it at a level that invites understanding. Unfortunately, I do not think I accomplished my goal. Most kids who would find the Lasky book interesting would still not be able to understand most of what I describe here, but I think this is a step in the right direction. My attempt at getting at the essence of ideas and trying to explain them in a kid-friendly way is very useful. I admire writers of children's books, because they think about this more than anyone else does.

The other thing I learned about was the relationship of mathematics and science. Separating the two disciplines deprives students of seeing how they are intimately linked. A friend of mine shared these thoughts when I asked him about the way math and science go together:

> Why do we have washboard (some say corduroy) dirt roads, should you vote in an election about whose candidates you know nothing, if light from the sun is parallel when it falls on the earth, how come it flares out when it shines through a hole in the clouds, why don't bugs have lungs, what does the weatherman mean when he says there's a 20% chance of rain, how can it be that the water level is going down when the tide is coming in, etc....? All these questions and thousands of others require, for their explanation, a bit of science married to a bit of mathematics. Like a poem you come to love even more after study, such understanding heightens your appreciation for the world around you and provides a never ending source of joy. In the process you come to treasure both.…
> *(Roger Pinkham, Professor of Mathematics, Stevens Institute of Technology, Hoboken, New Jersey)*

I hope what follows reflects this spirit.

Ancient Understandings of the World: Flat or Spherical?

When did people first realize that the earth might be round rather than flat? Apparently, Aristotle in the fourth century B.C. was convinced of the earth's sphericity, because he observed its shadow on the moon during a lunar eclipse. Less scholarly people such as sailors also thought the world might be round because they observed that the sails on ships approaching the horizon seemed to dip into the ocean.

But how round was the earth? No one had any idea, at least not until a curious and ingenious librarian named Eratosthenes (275–194 B.C.) discovered a remarkably simple method for measuring the circumference of something as large as the earth. He may have consid-

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ered three methods: (1) actual measurement, (2) measuring the earth's diameter, and (3) using trigonometry.

The first and most obvious way would be to circumnavigate the earth and keep track of the distance traveled. Traveling across land would be hard enough, but crossing oceans would be impossible, so Eratosthenes must have quickly ruled this possibility out.

In the second method, one could approximate the measurement indirectly by taking a trip through the center of the earth and measuring that distance. Then the fact that the circumference of the earth is a little more than three times its diameter could be used to make the approximation. But without a powerful machine that could cut such a tunnel, he surely shelved this idea as a child's fantasy.

In the third method, also an "indirect" measurement, one could find the circumference of a circle by measuring the *central angle* of the earth (ABC in Figure 1) and the arc distance (A to C).

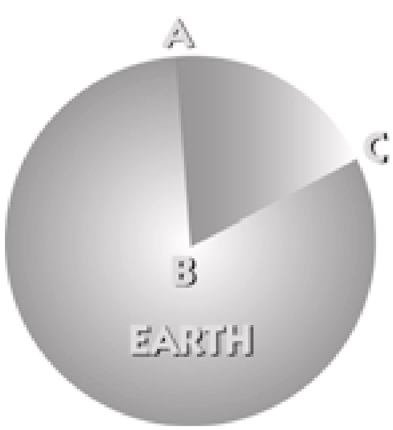

Figure 1.

Knowing there are 360 degrees in a circle helps us figure out how many ABC-sized "slices" would fit in the circle. If angle ABC is 60 degrees, then six slices would fit. That means six arcs (AC) would also fit around the circle, so all that we have to do is multiply the distance from A to C by 6 to find the circumference. That's easy enough. But we still have the same problem: How would Eratosthenes get these measurements? As it turns out, his brilliant insight was to study shadows!

Eratosthenes' Most Amazing Discovery: The "Sun" Angle and the Central Angle Are the Same!

One day Eratosthenes read in a papyrus book that at noon on June 21 in the frontier outpost town of Syene, vertical sticks cast no shadow and a reflection of the sun could be seen at the bottom of the well. As a scientist, he wanted to know if the same thing happened in Alexandria. He waited for June 21 and then discovered that a distinct shadow could be seen at noon in that city. But how could that be? How could the sun rays go directly down a well in Syene and at the same time there are shadows in a place approximately 500 miles away? The earth had to be round.

But how did this help him in measuring the central angle? Because Eratosthenes knew some geometry, he was able to come up with the answer.

In Figure 2, no shadow is seen at Syene while there is a shadow in Alexandria. Also, the "sun" angle in Alexandria is equal to the central angle, because they are alternate interior angles formed by a transversal line intersecting the sun's parallel rays. So, if the sun angle can be determined, then the central angle can be found when there is no shadow at the other site.

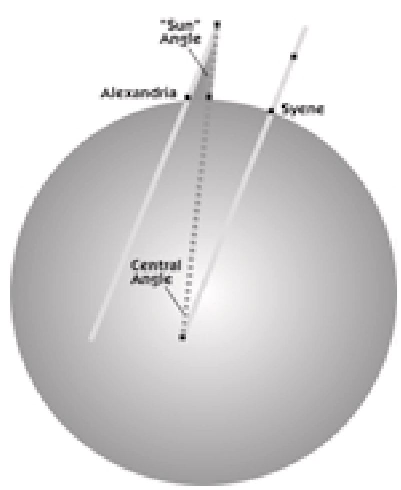

Figure 2.

But what happens if there are shadows at both sites? In Figure 3 we have modeled this problem with the Geometer's Sketchpad. Notice that neither sun angle A at Alexandria nor angle S at Syene equals the central angle.

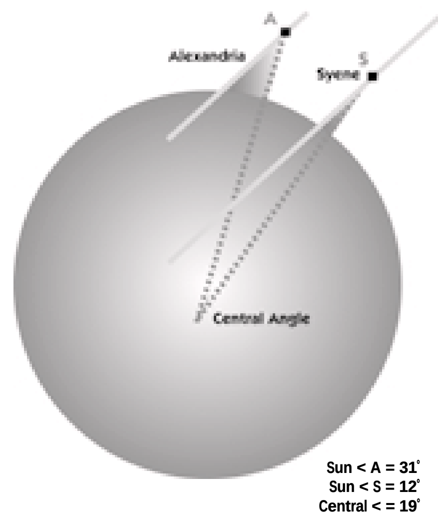

Figure 3.

The Great Discovery and Its Significance

If you study these diagrams, you will see that the central angle plus the sun angle at Syene equals the sun angle in Alexandria. Or expressed another way, the central angle equals the difference of the two sun angles. (For a proof of this, visit the Web site: http://k12science.stevens-tech.edu/noonday.)

This means that if you can measure the sun's angles at two different positions on the earth at the same time, then you can figure out the central angle! Let's look at an example to see how this information leads us to determine an empirical value for the circumference of the earth.

We'll choose two places that are fairly far apart on the earth's surface but still close enough that we can measure an acute angle: Manasquan, New Jersey, and San Juan, Puerto Rico.

Method 1: Using Scale Drawing

At local noon (that is, when the sun is at its highest point in the sky) on the same day, each set of experimenters measures the shadow cast by a meter stick.

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In Manasquan, the shadow's length measured 80.5 centimeters (cm). In San Juan, the shadow length was 35.3 cm (see Figure 4).

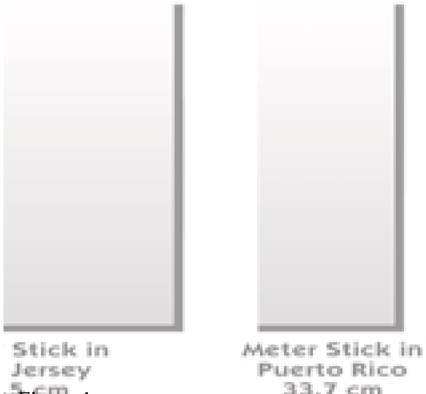

Figure 4.

The next step is to figure out the sun angles at Manasquan and Puerto Rico. A protractor can be used to measure these angles (see Figure 5).

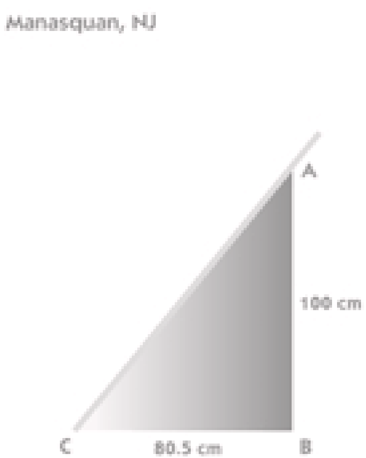

Figure 5.

Figure 6 shows how Geometer's Sketchpad can be used to measure the angles more accurately:

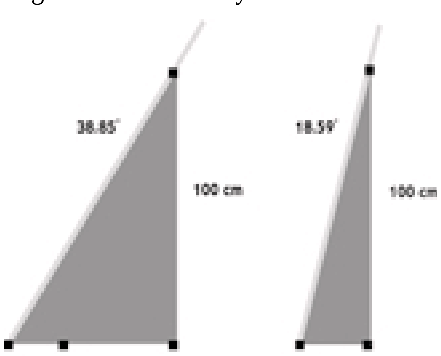

Figure 6.

The central angle equals the difference between these angles, which is 38.85 minus 18.59, or 20.26 degrees.

Now that we know the central angle, we can determine how many such angles would make up the full circle; that is, how many "slices" can we make that have an angle of 20.27 degrees? (See Figure 7.) Because the total circle is 360 degrees, you would have slightly fewer than 18 equal "slices" (17.77, to be more precise).

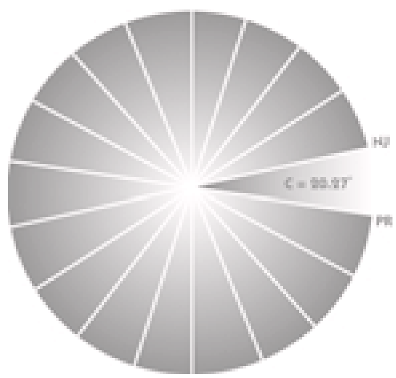

Figure 7.

Each slice's edge represents the distance between Manasquan and Puerto Rico, or, more accurately, the north-to-south distance between each site's line of latitude.

Next, we turn to the details of doing a collaborative project.

Manasquan, New Jersey is located at 40.13N 74.05W, while San Juan, Puerto Rico is at 18.47N 66.12W.

Notice that the latitudinal distance between these sites is 40.13 minus 18.47, or 21.66 degrees, which is approximately 2,404 kilometers (km), if we use 111 km for each degree of latitude. This can also be found by determining the scale of a world map and measuring. Of course, Eratosthenes did not have such luxuries as accurate maps, and he certainly did not know that each angle of latitude was approximately 111 km in length. But to understand this experiment, we will take some 20th-century liberties.

Based on the data given, the length of the "slice" is about 2,400 km. Because we have approximately 18 slices, the projected circumference is 2,400 times 18:

$2{,}400 \times 18 = 43{,}200$ km

This is off by about 8% when we compare it to the benchmark figure of 40,008 km for the earth's circumference.

This experiment can also be done using trigonometry. You will find that application at http://k12science.stevens-tech.edu/noonday/trig.html.

On March 21, 1998, you and your school can participate in a collaborative project with other schools and re-create Eratosthenes' measurement of the earth. Please visit the Web site at http://k12science.stevens-tech.edu/noonday for more information and updates.

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Figure 8.

Measuring the Earth Project

General Procedure. To do this experiment, you will need some materials to measure shadows accurately. You will use a *gnomon* to help your measurement: in this case, a meter stick perpendicular to the ground. For your measurements to be accurate, the meter stick *must* be vertical. (Note the device used by students at Manasquan High School in Figure 8.) Now guide your class through the following processes.

- Divide your class into working groups of three or four students per group.
- Set up your measurement station. Place paper under it so you can mark where the shadow ends. Because the edge of the shadow is "fuzzy" as it moves from west to east (in the northern hemisphere), tell students to place their marks carefully. They may find it interesting that the shadow points toward the north, but does it point to true or magnetic north? A compass will come in handy to determine this.
- Take measurements every two minutes beginning at least 10 minutes before local noon, which is the time when the sun is highest in the sky. Most likely, this will *not* be 12 noon as indicated on your watch. Your students should note that when the sun is highest in the sky, the shadow length is the shortest.
- After some discussion, each group reports its results to the entire class. Write each group's best value on the board. You and your class should assume that there will be different values, so your students will need to determine their "best" shadow length and decide which one will be the class's best estimate of the shadow length at local noon.
- Make a scale drawing of your stick and shadow. Complete the triangle and measure the sun's angle with a protractor.
- With your students, fill in Table 1.

  These values are submitted by e-mail or entered at a project Web site with the central database that contains information from all the schools that participate in the experiment. The data are then displayed at a Web site for everyone to see. (See the following Web site for more information on this process: http://k12science.stevens-tech.edu/noonday/collab.html.)
- Using this Web site, collect shadow length data from other locations. You can make a chart like Table 2 and place it next to a wall map of the world and then mark the locations of other participating schools.
- After you have several schools with entries, the students pick one of the schools to complete the chart. Have your students extend the chart to include the central angle, circumference, and percentage error. It should look like Table 3.
- Now that your students know the central angle, have them draw a picture of your school's location and the location of the other school on the circumference of a large circle.
- Ask the students to determine how many "slices" fit into this circle.
- Use the north–south distance to determine the value of the circumference. Explain why that particular measurement works.
- Calculate the percentage error and enter the results in Table 4. (Use 40,000 kilometers as your benchmark.)
- Repeat the same process for another school.
- An optional approach is to use trigonometry to determine the sun's angle. (Again, see the Web site for this article.)
- Connect with the Web site that has the calculations and see how well you and your class did. Confirm your calculations with the spreadsheet values listed there.
- Create a Web page that describes the activity and what you and students learned from it. Include a link to the Noon Day "quilt" designed by teachers and students at Faubion Middle School (http://www.geocities.com/Athens/8231/noonschs.htm); ask to have your Web page included in the quilt. ■

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References

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Table 1								
Site	*Latitude*	*Longitude*	*Shadow Length*	*Sun Angle*				
School Name								
Table 2								
Site	*Latitude*	*Longitude*	*Shadow Length*	*Sun Angle*				
Your School								
School 1								
School 2								
School 3								
etc.								
Table 3								
Site	*Latitude*	*Longitude*	*Shadow*	*Sun Angle Length*	*North–South Distance	Central Angle	Circumference	% Error
Your School								
Another School (from table above)								
Table 4								
Site	*Latitude*	*Longitude*	*Shadow*	*Sun Angle*	**North–South Length*	*Central Distance*	*Circumference Angle*	*% Error*
Your School								
Another School (from table above)								

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